CBSE Class 10 Mathematics

Important Questions

Chapter 12

Area Related to Circles

1 Marks Questions

Unless stated otherwise, take $\pi = \frac{22}{7}$.

1. The radii of two circles are 19 cm and 9 cm respectively. Find the radius of the circle which has circumference equal to the sum of the circumferences of the two circles.

Ans. Let R be the radius of the circle which has circumference equal to the sum of circumferences of the two circles, then according to question,

$$2\pi R = 2\pi(19) + 2\pi(9)$$

$\Rightarrow$ R = 19 + 9

$\Rightarrow$ R = 28 cm

2. The circumference of a circular field is 528 cm. Then its radius is

(a) 42 cm

(b) 84 cm

(c) 72 cm

(d) 56 cm

Ans. (b) 84 cm

3. The circumference of a circle exceeds its diameter by 180 cm. Then its radius is

(a) 32 cm

(b) 36 cm

(c) 40 cm

(d) 42 cm

Ans. (d) 42 cm

4. Area of the sector of angle 60° of a circle with radius 10 cm is

(a) $52\frac{5}{21} cm^2$

(b) $52\frac{8}{21} cm^2$

(c) $52\frac{4}{21} cm^2$

(d) none of there

Ans. (b) $52\frac{8}{21} cm^2$

5. Area of a sector of angle P of a circle with radius R is

(a) $\frac{P}{180} \times 2\pi R$

(b) $\frac{P}{180} \times \pi R^2$

(c) $\frac{P}{360} \times 2\pi R$

(d) $\frac{P}{720} \times 2\pi R^2$

Ans. (d) $\frac{P}{720}\times 2\pi R^2$

6. If the sum of the circumferences of two circles with radii R_1 and R_2 is equal to the circumference of a circle of Radius R_1, then

(a) $R_1 + R_2 = R$

(b) $R_1 + R_2 > R$

(c) $R_1 + R_2 < R$

(d) None of these

Ans. (A) $R_1 + R_2 = R$

7. If the perimeter of a circle is equal to that of a square, then the ratio of their area is

(a) 22:7

(b) 14:11

(c) 7:22

(d) 11:14

Ans. (c) 7:22

8. Area of a sector of angle p^0 of a circle with radius R is

(a) $\frac{P}{180}\times 2\pi R$

(b) $\frac{P}{180}\times \pi R^2$

(c) $\frac{P}{360}\times 2\pi R$

(d) $\frac{P}{720}\times 2\pi R^2$

Ans. d) $\frac{P}{720}\times 2\pi R^2$

9. The circumference of a circular field is 154 m. Then its radius is

(a) 7 **m**

(b) 14 m

(c) 7.5 m

(d) 28 cm

Ans. (a) 7 m

10. If the perimeter of a circle is equal to that of a square, then the ratio of their area is

(a) 22:7

(b) 14:11

(c) 7:22

(d) 11:14

Ans. (c) 7:22

11. Area of a sector of angle p° **of a circle with radius R is**

(a) $\frac{P}{180}\times 2\pi R$

(b) $\frac{P}{180} \times \pi R^2$

(c) $\frac{P}{360} \times 2\pi R$

(d) $\frac{P}{720} \times 2\pi R^2$

Ans. d) $\frac{P}{720} \times 2\pi R^2$

12. Area of the sector of angle $60°$ of a circle with radius 10 cm is

(a) $52\frac{5}{21} cm^2$

(b) $52\frac{8}{21} cm^2$

(c) $52\frac{4}{21} cm^2$

(d) none of these

Ans. (b) $52\frac{8}{21} cm^2$

13. The area of a circle is 394.24 cm^2. Then the radius of the circle is

(a) 11.4 cm

(b) 11.3 cm

(c) 11.2 cm

(d) 11.1 cm

Ans. (c) 11.2 cm

14. If the sum of the areas of two circles with radii R_1 and R_2 is equal to the area of a circle of radius R, then

(a) $R_1 + R_2 = R$

(b) $R_1^2 + R_2^2 = R^2$

(c) $R_1 + R_2 < R$

(d) $R^2_1 + R_2^2 < R^2$

Ans. (a) $R_1 + R_2 = R$

15. Circumference of a sector of angle P° of a circle with radius R is

(a) $\frac{P}{180} \times 2\pi R$

(b) $\frac{P}{180} \times \pi R^2$

(c) $\frac{P}{360} \times 2\pi R$

(d) $\frac{P}{720} \times 2\pi R^2$

Ans. (c) $\frac{P}{360} \times 2\pi R$

16. If the perimeter and area of circle are numerically equal, then the radius of the circle is

(a) 2 units

(b) π **units**

(c) 4 units

(d) 7 units

Ans. (a) 2 units

17. The radius of a circle is $\frac{7}{\sqrt{\pi}} cm$ **, then the area of the circle is**

(a) $154 cm^2$

(b) $\frac{49}{\pi} cm^2$

(c) 22 cm^2

(d) 49 cm^2

Ans. (d) 49 cm^2

18. Area of a sector of angle P° of a circle with radius R is

(a) $\frac{P}{180} \times 2\pi R$

(b) $\frac{P}{180} \times \pi R^2$

(c) $\frac{P}{360} \times 2\pi R$

(d) $\frac{P}{720} \times 2\pi R^2$

Ans. (a) $\frac{P}{180} \times 2\pi R$

19. The diameter of a circle whose area is equal to the sum of the area of the two circles of radii 24 cm and 7 cm is

(a) 31 cm

(b) 25 cm

(c) 62 cm

(d) 50 cm

Ans. (a) 31 cm

20. The circumference a circle is 528 cm. Then its area is

(a) 22,176 cm^2

(b) 22,576 cm^2

(c) 23,176 cm^2

(d) 24,576 cm^2

Ans. (a) 22,176 cm^2

CBSE Class 10 Mathematics

Important Questions

Chapter 12

Area Related to Circles

2 Marks Questions

1. The radii of two circles are 8 cm and 6 cm respectively. Find the radius of the circle having area equal to the sum of the areas of the two circles.

Ans. Let R be the radius of the circle which has area equal to the sum of areas of the two circles, then

According to the question,

$\pi R^2 = \pi(8)^2 + \pi(6)^2$

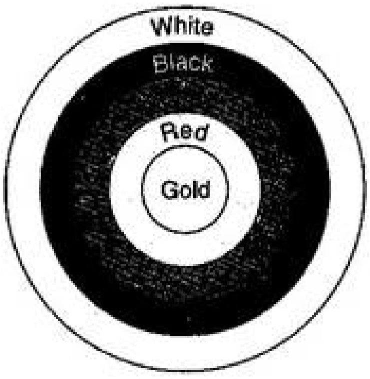
$\Rightarrow R^2 = (8)^2 + (6)^2$

$\Rightarrow R^2 = 64 + 36$

$\Rightarrow R^2 = 100$

$\Rightarrow R = 10$ cm

2. Figure depicts an archery target marked with its five scoring areas from the centre outwards as Gold, Red, Blue, Black and White. The diameter of the region representing Gold score is 21 cm and each of the other bands is 10.5 cm wide. Find the area of the five scoring regions.



Ans. Gold: Diameter = 21 cm

$\Rightarrow$ Radius = $\frac{21}{2}$ cm

Area of gold scoring region = $\pi\left(\frac{21}{2}\right)^2$ = $\frac{22}{7}\times\frac{21}{2}\times\frac{21}{2}$ = 346.5 cm^2

Red: Area of red scoring region = $\pi\left(\frac{21}{2}+10.5\right)^2-\pi\left(\frac{21}{2}\right)^2$ = $\pi(21)^2-346.5$

= $\frac{22}{7}\times21\times-346.5$ = 1386 – 346.5 = 1039.5 cm^2

Blue: Area of blue scoring region = $\pi(21+10.5)^2-(1039.5-346.5)$ = $\pi(31.5)^2-1386$

= $\frac{22}{7}\times31.5\times31.5-1386$ = 3118.5 – 1386 = 1732.5 cm^2

Black: Area of black scoring region = $\pi(31.5+10.5)^2-(1732.5+1039.5+346.5)$

= $\pi(42)^2-3118.5$ = $\frac{22}{7}\times42\times42-3118.5$

= 5544 – 3118.5 = 2425.5 cm^2

White: Area of white scoring region =
$\pi(42+10.5)^2-(2425.5+1732.5+1039.5+346.5)$

= $\pi(52.5)^2-5544$ = $\frac{22}{7}\times52.5\times52.5-5544$

= 8662.5 – 5544 = 3118.5 cm^2

3. The wheels of a car are of diameter 80 cm each. How many complete revolutions does each wheel make in 10 minutes when the car is travelling at a speed of 66 km per hour?

Ans. Diameter of wheel = 80 cm

$\Rightarrow$ Radius of wheel (r) = 40 cm

Distance covered by wheel in one revolution = $2\pi r$ = $2\times\frac{22}{7}\times 40$ = $\frac{1760}{7}$ cm

$\because$ Distance covered by wheel in 1 hour = 66 km = 66000 m = 6600000 cm

$\therefore$ Distance covered by wheel in 10 minutes = $\frac{6600000}{60}\times 10$ = 1100000 cm

$\therefore$ No. of revolutions = $\frac{\text{Total distance}}{\text{distance of one revolution}}$ = $\frac{1100000\times 7}{1760}$ = 4375

4. Tick the correct answer in the following and justify your choice: If the perimeter and area of a circle are numerically equal, then the radius of the circle is:

(A) 2 units

(B) π units

(C) 4 units

(D) 7 units

Ans. (A) Circumference = Area

$\Rightarrow 2\pi r = \pi r^2$

$\Rightarrow r$ = 2 units

Unless stated otherwise, take $\pi = \frac{22}{7}$.

5. Find the area of a sector of a circle with radius 6 cm, if angle of the sector is 60°.

Ans. Here, r = 6 cm and $\theta = 60^\circ$

Area of sector = $\frac{\theta}{360^\circ}\times\pi r^2$

$= \frac{60^\circ}{360^\circ} \times \frac{22}{7} \times 6 \times 6$

$= \frac{132}{7}$ cm^2

6. Find the area of a quadrant of a circle whose circumference is 22 cm.

Ans. Given, $2\pi r$ = 22 $\Rightarrow 2 \times \frac{22}{7} \times r = 22 \Rightarrow r = \frac{7}{2}$ cm

We know that for quadrant of circle, $\theta = 90^\circ$

$\therefore$ Area of quadrant = $\frac{\theta}{360^\circ} \times \pi r^2$

$= \frac{90^\circ}{360^\circ} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}$

$= \frac{77}{8}$ cm^2

7. The length of the minute hand of a clock is 14 cm. Find the area swept by the minute hand in 5 minutes.

Ans. Here, r = 14 cm and $\theta = \frac{90^\circ}{3} = 30^\circ$

$\therefore$ Area swept = $\frac{\theta}{360^\circ} \times \pi r^2$

$= \frac{30^\circ}{360^\circ} \times \frac{22}{7} \times 14 \times 14$

$= \frac{154}{3}$ cm^2

8. A chord of a circle of radius 10 cm subtends a right angle at the centre. Find the area

of the corresponding: (i) minor segment, (ii) major segment. $\left(\text{Use } \pi = 3.14\right)$

Ans. i) Here, r = 10 cm and $\theta = 90^\circ$

$\therefore$ Area of minor sector = $\frac{\theta}{360^\circ} \times \pi r^2$

= $\frac{90^\circ}{360^\circ} \times 3.14 \times 10 \times 10$ = 78.5 cm^2

Area of ΔOAB = $\frac{1}{2}$ x Base x Height

= $\frac{1}{2} \times 10 \times 10$ = 50 cm^2

$\therefore$ Area of minor segment = Area of minor sector – Area of ΔOAB

= 78.5 – 50 = 28.5 cm^2

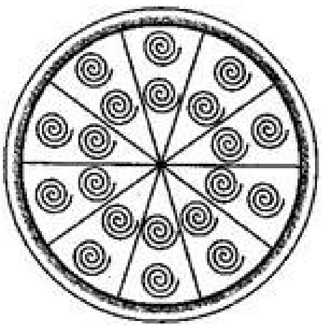
(ii) For major sector, radius = 10 cm and $\theta = 360^\circ - 90^\circ = 270^\circ$

$\therefore$ Area of major sector = $\frac{\theta}{360^\circ} \times \pi r^2$ = $\frac{270^\circ}{360^\circ} \times 3.14 \times 10 \times 10$ = 235.5 cm^2

9. A horse is tied to a peg at one corner of a square shaped grass field of side 15 m by means of a 5 m long rope (see figure). Find:

(i) the area of that part of the field in which the horse can graze.

(ii) the increase in the grazing area if the rope were 10 m long instead of 5 cm. $\left(\text{Use } \pi = 3.14\right)$



Ans. (i) Area of quadrant with 5 m rope = $\frac{\theta}{360^\circ} \times \pi r^2$ = $\frac{90^\circ}{360^\circ} \times 3.14 \times 5 \times 5$ = 19.625 m^2

(ii) Area of quadrant with 10 m rope = $\frac{\theta}{360^\circ}\times\pi r^2$ = $\frac{90^\circ}{360^\circ}\times 3.14\times 10\times 10$ = 78.5 m^2

$\therefore$ The increase in grazing area = 78.5 – 19.625 = 58.875 m^2

10. A brooch is made with silver wire in the form of a circle with diameter 35 mm. The wire is also used in making 5 diameters which divide the circle into 10 equal sectors as shown in figure. Find:

(i) the total length of the silver wire required.

(ii) the area of each sector of the brooch.

Ans. (i) Diameter = 35 mm

$\Rightarrow$ Radius = $\frac{35}{2}$ mm

Circumference = $2\pi r$ = $2\times\frac{22}{7}\times\frac{35}{2}$ = 110 mm ....(i)

Length of 5 diameters = 35 x 5 = 175 mm ....(ii)

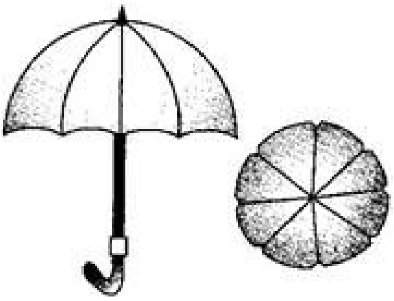
$\therefore$ Total length of the silver wire required = 110 + 175 = 285 mm

(ii) $r=\frac{35}{2}$ mm and $\theta=\frac{360^\circ}{10}=36^\circ$

$\therefore$ The area of each sector of the brooch = $\frac{\theta}{360^\circ}\times\pi r^2$

= $\frac{36^\circ}{360^\circ}\times\frac{22}{7}\times\frac{35}{2}\times\frac{35}{2}$ = $\frac{385}{4}$ mm^2

11. An umbrella has 8 ribs which are equally spaced (see figure). Assuming umbrella to be a flat circle of radius 45 cm, find the area between the two consecutive ribs of the umbrella.



Ans. Here, r = 45 cm and $\theta = \frac{360^\circ}{8} = 45^\circ$

Area between two consecutive ribs of the umbrella = $\frac{\theta}{360^\circ} \times \pi r^2$

= $\frac{45^\circ}{360^\circ} \times \frac{22}{7} \times 45 \times 45$ = $\frac{22275}{28}$ cm^2

12. A car has two wipers which do not overlap. Each wiper has a blade of length 25 cm sweeping through an angle of 115°. Find the total area cleaned at each sweep of the blades.

Ans. Here, r = 25 cm and $\theta = 115^\circ$

The total area cleaned at each sweep of the blades = $2 \times \left(\frac{\theta}{360^\circ} \times \pi r^2 \right)$

= $2 \times \left(\frac{115^\circ}{360^\circ} \times \frac{22}{7} \times 25 \times 25 \right)$ = $\frac{158125}{126}$ cm^2

13. To warn ships for underwater rocks, a lighthouse spreads a red coloured light over a sector of angle 80° to a distance of 16.5 km. Find the area of the sea over which the ships are warned. $(\text{Use } \pi = 3.14)$

Ans. Here, r = 16.5 km and $\theta = 80^\circ$

The area of sea over which the ships are warned = $\frac{\theta}{360^\circ} \times \pi r^2$

$= \frac{80°}{360°} \times 3.14 \times 16.5 \times 16.5$ = 189.97 km^2

14. Tick the correct answer in the following:

Area of a sector of angle p (in degrees) of a circle with radius R is:

(A) $\frac{p}{180°} \times 2\pi r$

(B) $\frac{p}{180°} \times \pi r^2$

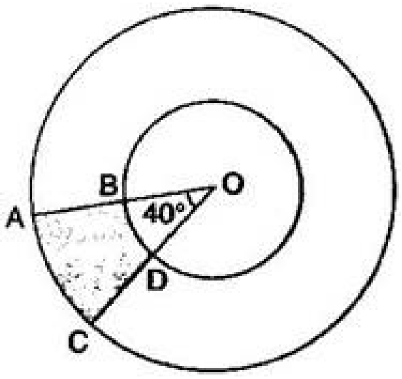
(C) $\frac{p}{360°} \times 2\pi r$

(D) $\frac{p}{360°} \times 2\pi r^2$

Ans. (D) Given, r = R and $\theta = p$

Area of sector = $\frac{\theta}{360^\circ} \times \pi r^2 = \frac{p}{360^\circ} \times \pi R^2 = \frac{p}{2 \times 360^\circ} \times 2\pi R^2 = \frac{p}{720^\circ} \times 2\pi R^2$

15. Find the area of the shaded region in figure, if radii of the two concentric circles with centre O are 7 cm and 14 cm respectively and $\angle$ AOC = $40°$.



Ans. Area of shaded region = Area of sector OAC – Area of sector OBD

$= \frac{40°}{360°} \times \frac{22}{7} \times (14)^2 - \frac{40°}{360°} \times \frac{22}{7} \times (7)^2$

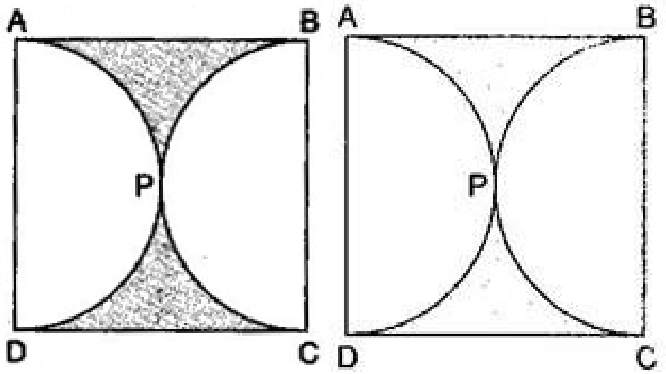
$$= \frac{40^\circ}{360^\circ} \times \frac{22}{7}\left[(14)^2 - (7)^2\right]$$

$$= \frac{40^\circ}{360^\circ} \times \frac{22}{7}(14-7)(14+7)$$

$$= \frac{40^\circ}{360^\circ} \times \frac{22}{7} \times 7 \times 21$$

$$= \frac{154}{3}\,\text{cm}^2$$

16. Find the area of the shaded region in figure, if ABCD is a square of side 14 cm and APD and BPC are semicircles.



Ans. Area of shaded region

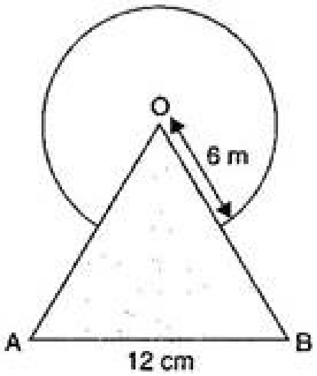
= Area of square ABCD – (Area of semicircle APD + Area of semicircle BPC)

$$= 14 \times 14 - \left[\frac{1}{2} \times \frac{22}{7}\left(\frac{14}{2}\right)^2 + \frac{1}{2} \times \frac{22}{7}\left(\frac{14}{2}\right)^2\right]$$

$$= 196 - \frac{22}{7} \times 7 \times 7$$

= 196 – 154 = 42 cm^2

17. Find the area of the shaded region in figure, where a circular arc of radius 6 cm has been drawn with vertex O of an equilateral triangle OAB of side 12 cm as centre.



Ans. Area of shaded region

= Area of circle + Area of equilateral triangle OAB – Area common to the circle and the triangle

= $\pi(6)^2 + \frac{\sqrt{3}}{4}(12)^2 - \frac{60^\circ}{360^\circ} \times \pi(6)^2$

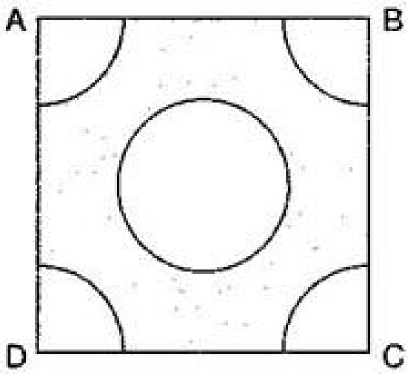
= $36\pi + 36\sqrt{3} - 6\pi$

= $30\pi + 36\sqrt{3}$

= $30 \times \frac{22}{7} + 36\sqrt{3}$

= $\left(\frac{660}{7} + 36\sqrt{3}\right)$ cm^2

18. From each corner of a square of side 4 cm a quadrant of a circle of radius 1 cm is cut and also a circle of diameter 2 cm is cut as shown in figure. Find the area of the remaining portion of the figure.



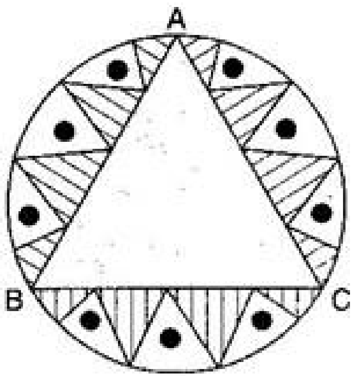
Ans. Area of remaining portion of the square

= Area of square – (4 x Area of a quadrant + Area of a circle)

$$= 4\times4-\left[4\times\frac{90^\circ}{360^\circ}\times\frac{22}{7}\times(1)^2+\frac{22}{7}\times\left(\frac{2}{2}\right)^2\right]$$

$$= 16-2\times\frac{22}{7} = \frac{68}{7}\text{ cm}^2$$

19. In a circular table cover of radius 32 cm, a design is formed leaving an equilateral triangle ABC in the middle as shown in figure. Find the area of the design (shaded region).

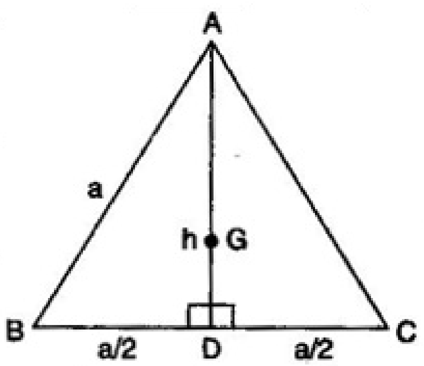


Ans. Area of design = Area of circular table cover – Area of the equilateral triangle ABC

$$= \pi(32)^2-\frac{\sqrt{3}}{4}a^2 \text{ ....(i)}$$

∵ G is the centroid of the equilateral triangle.

∴ radius of the circumscribed circle = $\frac{2}{3}h$ cm



According to the question, $\frac{2}{3}h=32$

$\Rightarrow h$ = 48 cm

Now, $a^2 = h^2 + \left(\frac{a}{2}\right)^2$

$\Rightarrow a^2 = h^2 + \frac{a^2}{4}$

$\Rightarrow a^2 - \frac{a^2}{4} = h^2$

$\Rightarrow \frac{3a^2}{4} = h^2$

$\Rightarrow a^2 = \frac{4h^2}{3}$

$\Rightarrow a^2 = \frac{4(48)^2}{3}$ = 3072

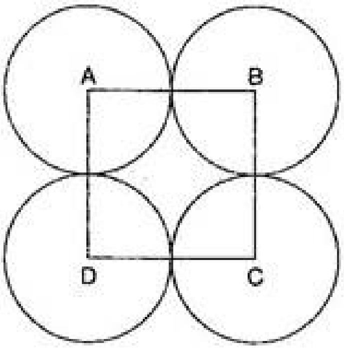
$\Rightarrow a = \sqrt{3072}$

$\therefore$ Required area = $\pi(32)^2 - \frac{\sqrt{3}}{4} \times 3072$ [From eq. (i)]

= $\frac{22}{7} \times 1024 - 768\sqrt{3}$

= $\left(\frac{22528}{7} - 768\sqrt{3}\right)$ cm^2

20. In figure ABCD is a square of side 14 cm. With centers A, B, C and D, four circles are drawn such that each circle touches externally two of the remaining three circles. Find the area of the shaded region.

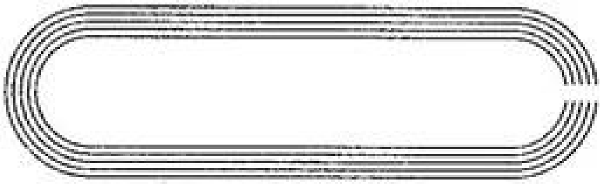


Ans. Area of shaded region = Area of square – 4 x Area of sector

$= 14 \times 14 - 4 \times \frac{90^\circ}{360^\circ} \times \frac{22}{7} \times \left(\frac{14}{2}\right)^2$

$= 196 - \frac{22}{7} \times 7 \times 7$ = 196 – 154 = 42 cm^2

21. Figure depicts a racing track whose left and right ends are semicircular.



Ans. (i) Distance around the track along its inner edge

$= 106 + 106 + 2 \times \left[\frac{180^\circ}{360^\circ} \times 2 \times \frac{22}{7} \times \left(\frac{60}{2}\right)\right]$

$= 212 + 60 \times \frac{22}{7} = 212 + \frac{1320}{7} = \frac{2804}{7}$ m

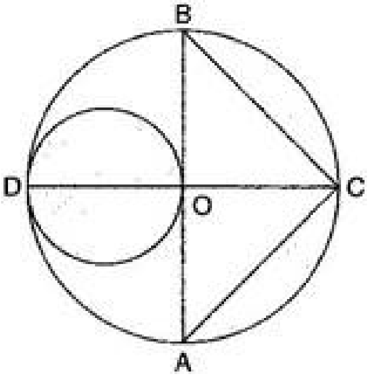
(ii) Area of track $= 106 \times 10 + 106 \times 10 + 2 \times \left[\frac{1}{2} \times \frac{22}{7}(30+10)^2 - \frac{1}{2} \times \frac{22}{7}(30)^2\right]$

$= 1060 + 1060 + \frac{22}{7}\left[(40)^2 - (30)^2\right]$

$= 2120 + \frac{22}{7} \times 700$

$= 4320 \text{ m}^2$

22. In figure, AB and CD are two diameters of a circle (with centre O) perpendicular to each other and OD is the diameter of the smaller circle. If OA = 7 cm, find the area of the shaded region.

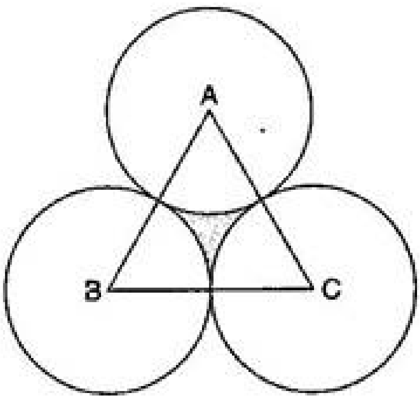


Ans. Area of shaded region = Area of circle + Area of semicircle ACB – Area of $\triangle$ACB

$$= \frac{22}{7}\times\left(\frac{7}{2}\right)^2 + \frac{1}{2}\times\frac{22}{7}\times(7)^2 - \left(\frac{7\times 7}{2}+\frac{7\times 7}{2}\right)$$

$$= \frac{77}{2}+187-49 = \frac{133}{2} = 66.5 \text{ cm}^2$$

23. The area of an equilateral triangle ABC is 17320.5 cm². With each vertex of the triangle as centre, a circle is drawn with radius equal to half the length of the side of the triangle (see figure). Find the area of the shaded region. $\left(\text{Use } \pi = 3.14 \text{ and } \sqrt{3} = 1.73205\right)$



Ans. Area of equilateral triangle = $\frac{\sqrt{3}}{4}a^2$ = 17320.5

$$\Rightarrow a^2 = \frac{17320.5 \times 4}{\sqrt{3}}$$

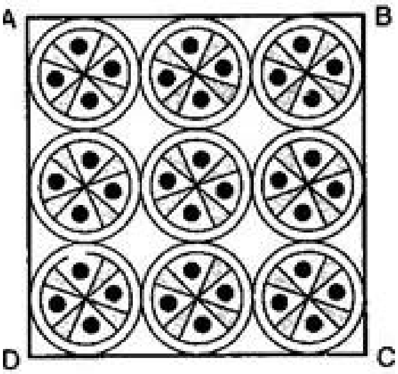
$$\Rightarrow a^2 = \frac{17320.5 \times 4}{1.73205}$$

$$\Rightarrow a^2 = 40000$$

$$\Rightarrow a = 200 \text{ cm}$$

Area of shaded region = Area of ΔABC $- 3\left[\frac{60^\circ}{360^\circ} \times 3.14 \times \left(\frac{200}{2}\right)^2\right]$

24. On a square handkerchief, nine circular designs each of radius 7 cm are made (see figure). Find the area of the remaining portion of the handkerchief.



Ans. Area of remaining portion of handkerchief = Area of square ABCD – Area of 9 circular designs

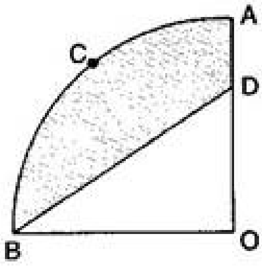
$= 42 \times 42 - 9 \times \frac{22}{7} \times 7 \times 7$

= 1764 – 1386 = 378 cm^2

25. In figure, OACB is a quadrant of a circle with centre O and radius 3.5 cm. If OD = 2 cm, find the area of the:

(i) quadrant OACB

(ii) shaded region



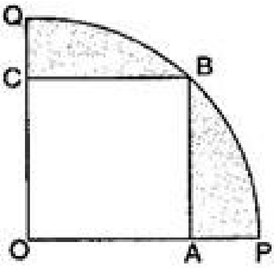
Ans. (i) Area of quadrant OACB = $\frac{90^\circ}{360^\circ} \times \frac{22}{7} \times (3.5)^2$

$= \frac{1}{4} \times \frac{22}{7} \times \frac{35}{10} \times \frac{35}{10} = \frac{77}{8}$ cm^2

(ii) Area of shaded region = Area of quadrant OACB – Area of ΔOBD

$= \frac{77}{8} - \frac{OB \times OD}{2} = \frac{77}{8} - \frac{3.5 \times 2}{2} = \frac{77}{8} - \frac{35}{10} = \frac{49}{8}$ cm^2

26. In figure, a square OABC is inscribed in a quadrant OPBQ. If OA = 20 cm, find the area of the shaded region. $(\text{Use } \pi = 3.14)$



Ans. OB = $\sqrt{OA^2 + AB^2}$ = $\sqrt{OA^2 + OA^2}$

= $\sqrt{2}$ OA = $\sqrt{2} \times 20 = 20\sqrt{2}$ cm

Area of shaded region = Area of quadrant OPBQ – Area of square OABC

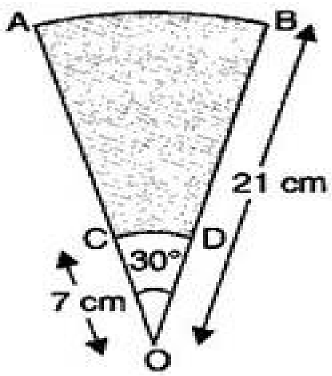
$= \frac{90^\circ}{360^\circ} \times 3.14 \left(20\sqrt{2}\right)^2 - 20 \times 20$

$= \frac{1}{4} \times 3.14 \times 800 - 400$

= 200 x 3.14 – 400

$= 228\ cm^2$

27. AB and CD are respectively arcs of two concentric circles of radii 21 cm and 7 cm and centre O (see figure). If $\angle AOB = 30^\circ$, find the area of the shaded region.



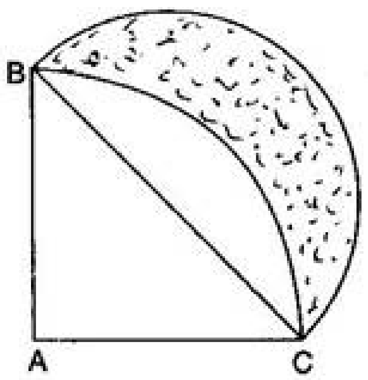
Ans. Area of shaded region = Area of sector OAB – Area of sector OCD

$= \frac{30^\circ}{360^\circ} \times \frac{22}{7} \times 21 \times 21 - \frac{30^\circ}{360^\circ} \times \frac{22}{7} \times 7 \times 7$

$= \frac{1}{12} \times \frac{22}{7} \times 21 \times 21 - \frac{1}{12} \times \frac{22}{7} \times 7 \times 7$

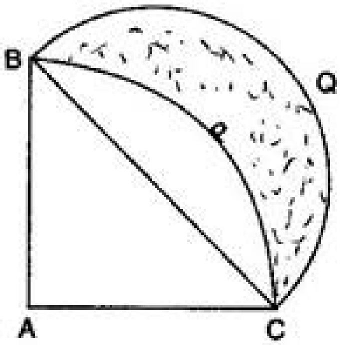
$= \frac{231}{2} - \frac{77}{6} = \frac{692 - 77}{6} = \frac{616}{6} = \frac{308}{3}\ cm^2$

28. In figure, ABC is a quadrant of a circle of radius 14 cm and a semicircle is drawn with BC as diameter. Find the area of the shaded region.



Ans. In right triangle BAC, $BC^2 = AB^2 + AC^2$[Pythagoras theorem]

$\Rightarrow BC^2 = (14)^2 + (14)^2 = 2(14)^2 \Rightarrow BC = 14\sqrt{2}$ cm



$\therefore$ Radius of the semicircle = $\frac{14\sqrt{2}}{2} = 7\sqrt{2}$ cm

$\therefore$ Required area = Area BPCQB

= Area BCQB – Area BCPB

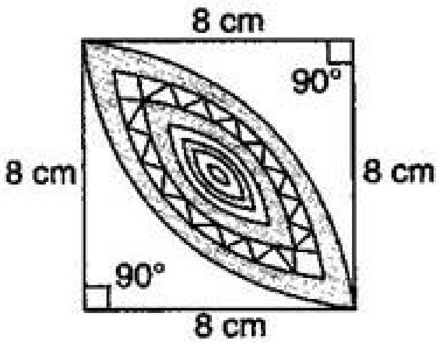
= Area BCQB – (Area BACPB – Area ΔBAC)

$$= \frac{180^\circ}{360^\circ} \times \frac{22}{7}\left(7\sqrt{2}\right)^2 - \left[\frac{90^\circ}{360^\circ} \times \frac{22}{7} \times (14)^2 - \frac{14 \times 14}{2}\right]$$

$$= \frac{1}{2} \times \frac{22}{7} \times 98 - \left(\frac{1}{4} \times \frac{22}{7} \times 196 - 98\right)$$

= 154 – (154 – 98) = 98 cm^2

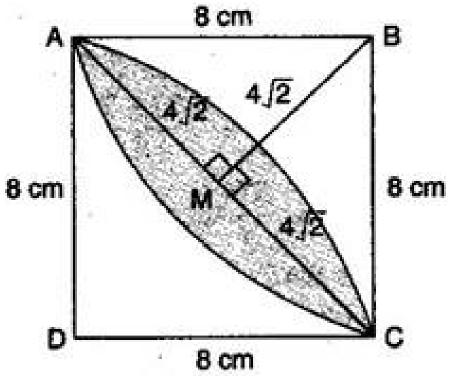
29. Calculate the area of the designed region in figure common between the two quadrants of circles of radius 8 cm each.



Ans. In right triangle ADC, $AC^2 = AD^2 + CD^2$ [Pythagoras theorem]

$\Rightarrow AC^2 = (8)^2 + (8)^2 = 2(8)^2$

$\Rightarrow$ AC = $\sqrt{128}$ = $8\sqrt{2}$ cm



Draw BM $\perp$ AC.

Then AM = MC = $\frac{1}{2}$ AC

= $\frac{1}{2} \times 8\sqrt{2}$ = $4\sqrt{2}$ cm

In right triangle AMB,

$AB^2 = AM^2 + BM^2$ [Pythagoras theorem]

$\Rightarrow (8)^2 = (4\sqrt{2})^2 + BM^2$

$\Rightarrow BM^2 = 64 - 32 = 32$

$\Rightarrow$ BM = $4\sqrt{2}$ cm

$\therefore$ Area of ΔABC = $\frac{1}{2}$ x AC x BM

= $\frac{8\sqrt{2} \times 4\sqrt{2}}{2}$ = 32 cm^2

$\therefore$ Half Area of shaded region = $\frac{90^\circ}{360^\circ} \times \frac{22}{7} \times (8)^2 - 32$

= $16 \times \frac{22}{7} - 32$ = $\frac{128}{7} cm^2$

$\therefore$ Area of designed region = 2 x $\frac{128}{7}$ = $\frac{256}{7}$ cm^2

---

30. Find the circumference of a circle of diameter 14cm.

Ans. $d = 14cm$

$\therefore r = 7cm$

Circumference $= 2\pi r$

$= 2\times\frac{22}{7}\times 7 = 44cm$

31. The diameter of a circular pond is 17.5m. It is surrounded by a path of width 3.5m. Find the area of the path.

Ans. Diameter of pond = 17.5cm

R = 8.75m

Width = 3.5m

Outer radius = R = 8.75 + 3.5 = 12.25m

Now,

Area of path $= \pi\left[R^2 - r^2\right]$

$= \pi\left[R + r\right)\left(R - r\right)$

$= \frac{22}{7}\left[12.25 + 8.75\right)\left(12.25 - 8.75\right)$

$= \frac{22}{7}\times 21\times 3.50 = \frac{66\times 7}{2}$

$= 33\times 7 = 231m^2$

32. Find the area of a sector of a circle with radius 6cm, if angle of the sector is 60°.

Ans. Diameter of pond = 17.5cm

R = 8.75m

Width = 3.5m

Outer radius = R = 8.75 + 3.5 = 12.25m

Now,

Area of path $= \pi\left[R^2 - r^2\right]$

$= \pi\left[R + r\right)\left(R - r\right)$

$= \frac{22}{7}\left[12.25 + 8.75\right)\left(12.25 - 8.75\right)$

$= \frac{22}{7} \times 21 \times 3.50 = \frac{66 \times 7}{2}$

$= 33 \times 7 = 231 m^2$

33. Find the area of a quadrant of a circle whose circumference is 22 cm.

Ans. Circumference = 22 cm

$2\pi r = 22$

$\Rightarrow r = \frac{22}{2\pi} = \frac{11}{\pi}$

Quadrant of circle will subtend 90° angle at the centre of the circle.

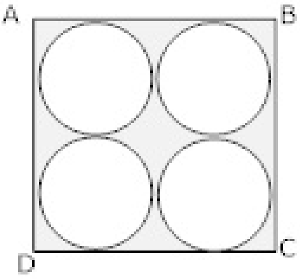
Area of such quadrant of the circle $= \frac{90°}{360°} \times \pi \times r^2$

$= \frac{1}{4\pi} \times \pi \times \left(\frac{11}{\pi}\right)^2$

$= \frac{121}{4\pi} = \frac{121 \times 7}{4 \times 22}$

$= \frac{77}{8} cm^2$

34. Find the area of the shaded region where ABCD is a square of side 14cm.



Ans. Area of square $ABCD = 14 \times 14 cm^2 = 196 cm^2$

Diameter of each circle $= \frac{14}{2} = 7cm$

∴ Radius of each circle $= \frac{7}{2} cm$

Area of each circle = $\pi r^2 = \frac{22}{7} \times \left(\frac{7}{2}\right)^2$

Area of 4 circles = $4 \times \frac{22}{7} \times \left(\frac{7}{2}\right)^2 = 154\ cm^2$

Area of shaded region = Area of square – Area of 4 circles

= 196 cm^2 – 154 cm^2 = 42 cm^2

35. The radius of a radius of a circle is 20cm. Three more concentric circles are drawn inside it in such a manner that it is divided into four parts of equal area. Find the radius of the largest of the three concentric circles.

Ans. Let r be the radius of the largest of the three circles

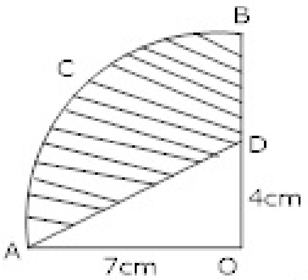
Area of largest circle $= \frac{3}{4}$ [area of given circle]

$$\therefore \pi r^2 = \frac{3}{4}\pi(20)^2$$

$$\Rightarrow r^2 = 300$$

$$\Rightarrow r = \sqrt{300} = 10\sqrt{3}$$

36. OACB is a quadrant of a circle with centre O and radius 7 cm. If OD = 4cm, then find area of shaded region.



Ans. Area of quadrant $OACB = \frac{90}{360}\pi(7)^2$

$$= \frac{49}{4} \times \frac{22}{7} = \frac{77}{2} cm^2$$

$\therefore$ Area of shaded region = Area of quadrant $OACB - area$ of ΔOAD

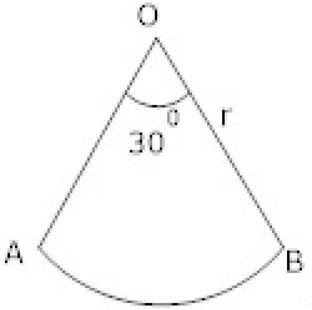
$$= \frac{77}{2} - \frac{1}{2}(7 \times 4) = \frac{77}{2} - 14 = \frac{49}{2}$$

$$= 24.5 cm^2$$

37. A pendulum swings through on angle of $30°$ and describes an arc 8.8cm in length. Find the length of pendulum.

Ans. Let r be the length of pendulum

$\angle AOB = 30° = \frac{\pi}{180°} \times 30° = \frac{\pi}{6}$



$\theta = \frac{l}{r}$

$\Rightarrow \frac{\pi}{6} = \frac{8.8}{r}$

$\Rightarrow r = \frac{8.8 \times 6}{\pi}$

$= 16.8\ cm$

38. The cost of fencing a circular field at the rate of Rs. 24 per metre is Rs. 5280. The field is to be ploughed at the rate of Rs.0.50per m^2. Find the cost of ploughing the field.

(Take $r = \frac{22}{7}$ **)**

Ans. Since for Rs. 24, the length of fencing = 1metre

$\therefore$ for Rs.5280, the length fencing $= \frac{1}{24} \times 5280 = 220$ meters

$\therefore$ Perimeter i.e., circumference of the field = 220meters

Let r be the radius of the field

$\therefore 2\pi r = 220$

$\Rightarrow r = \frac{220 \times 7}{2 \times 22} = 35\ m$

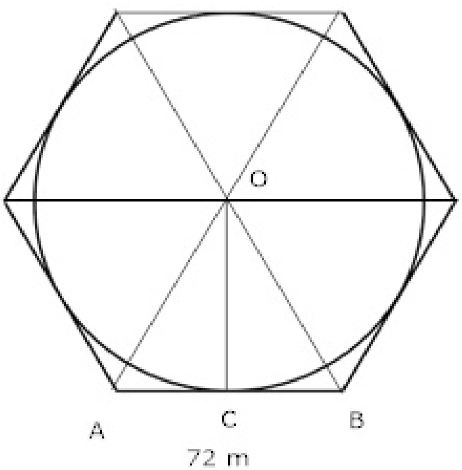
$\therefore$ Area of the field $= \pi r^2 = \pi(35)^2 = 1225\pi m^2$

Rate = Rs.0.50 per m^2

$\therefore$ Total cost of ploughing the field = Rs. $(1225\pi \times 0.50) = Rs.\frac{1225 \times 22 \times 1}{7 \times 2}$

$= Rs.(175 \times 11) = Rs.1925$

39. Find the difference between the area of regular hexagonal plot each of whose side 72m and the area of the circular swimming take in scribed in it. (Take $r = \frac{22}{7}$)



Ans. Side of hexagonal plot = 72m

Area of equilateral triangle $OAB = \frac{\sqrt{3}}{4}(side)^2 = \frac{\sqrt{3}}{4}(72)^2 = 1296\sqrt{3}m^2$

$\therefore$ Area of hexagonal plot = $6 \times$ Area of triangle OAB

$= 6 \times 1296\sqrt{3} = 7776(1.732)$

$= 13468.032m^2$

$OC^2 = OA^2 - AC^2 = (72)^2 - \left(\frac{72}{2}\right)^2$

$= 5184 - 1296 = 3888$

$OC^2 = 3888$

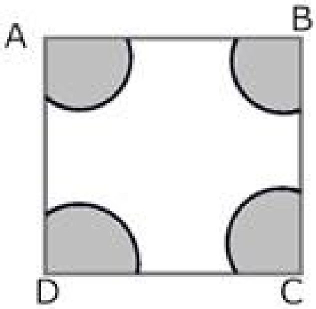
$\Rightarrow OC = \sqrt{3888} = 62\, m$

Area of circular region = $\pi r^2 = \frac{22}{7} \times (62)^2 = 12081\, m^2$

Difference = 13468 m^2 – 12081 m^2 = 1385 m^2

40. In the given figure areas have been drawn of radius 21cm each with vertices A,B,C

and D of quadrilateral ABCD as centers. Find the area of the shaded region.



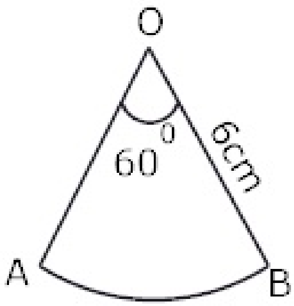
Ans. Required area = Area of the circle with radius 21

$= \pi(21)^2$

$= \frac{22}{7} \times 21 \times 2cm$

$= 22 \times 63 = 1386cm^2$

41. Find the area of a sector of a circle with radius 6cm, if angle of the sector is $60°$.



Ans. We know that Area of sector = $\frac{\theta}{360}\pi r^2$

$Here,\ \theta = 60, r = 6$

$\therefore$ Required area $= \frac{60}{360} \times \frac{22}{7} \times (6)^2$

$= \frac{1}{6} \times \frac{22}{7} \times 36$

$= \frac{22 \times 6}{7}$

$= \frac{132}{7} = 18\frac{6}{7}cm^2$

42. A wheel has diameter 84cm, find how many complete revolutions it must make to complete 792 meters.

Ans. Diameter, 2r = 84cm

Distance covered in one revolution= circumference = $2\pi r=\pi(2r)=\frac{22}{7}\times 84$

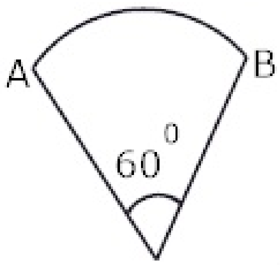
$=22\times 12=264\ cm$

Thus, for distance covered 264cm, number of revolution = 1

$\therefore$ For distance covered 792 metres = 79200cm

$\therefore$ Number of revolutions = $\frac{1}{264}\times 79200=300$

43. The given figure is a sector of a circle of radius 10.5cm. Find the perimeter of the sector. (Take $\pi=\frac{22}{7}$)



Ans. We know that circumference i.e, perimeter of a sector of angle P° of a circle with radius R

$=\frac{P^\circ}{360^\circ}\times 2\pi R+2r$

$\therefore$ Required perimeter

$=\frac{60^\circ}{360^\circ}\times\frac{2\times 22}{7}\times(10.5)+2(10.5)$

$=\frac{1}{6}\times\frac{44}{7}\times\frac{21}{2}+21$

$=32\ cm$

44. A car had two wipers which do not overlap. Each wiper has a blade of length 25cm sweeping through an angle of 115°. Find the total area cleaned at each sweep of the blades.

Ans. Radius of each wiper = 25cm, Angle = 115°

$\therefore \theta = 115°$

Total area cleaned at each sweep of the blades

$$= 2\left(\frac{115}{360} \times \frac{22}{7} \times 25 \times 25\right)\left[Area = \frac{\theta}{360} \times \pi r^2\right]$$

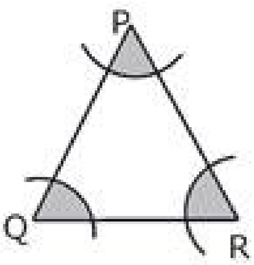
$$= \frac{230 \times 22 \times 5 \times 25}{72 \times 7} = \frac{230 \times 11 \times 125}{36 \times 7}$$

$$= \frac{115 \times 11 \times 125}{18 \times 7}$$

$$= \frac{158125}{126} cm^2$$

$$= 1254.96\ cm^2$$

45. In the given figure arcs have been drawn with radii 14 cm each and with centres P, Q and R. Find the area of the shaped region.



Ans. Area of shaded region

$$= \frac{\pi r^2}{360°} \times 180°$$

$$= \frac{22}{7} \times \frac{(14)^2}{2} = 11 \times 2 \times 14$$

$= 22 \times 14$

$= 308\ cm^2$

46. The radii of two circles are 19cm and 9cm respectively. Find the radius of the circle which has its circumference equal to the sum of the circumference of the two circles

Ans. C_1 = circumference of the 1st circle = 2π (19) = 38π cm

C_2 = circumference of the 2nd circle = 2π (19) = 18π cm

$$C_1 - C_2 = 38\pi + 18\pi$$
$$= 56\pi\ cm$$

47. A car travels 0.99km distance in which each wheel makes 450 complete revolutions. Find the radius of its wheel

Ans. Distance travelled by a wheel in 450 complete revolutions = 0.99 km = 990m

Distance travelled in one revolution = $\frac{990}{450} = \frac{11}{5} m$

Let r be the radius of the wheel

$\therefore 2\pi r = \frac{11}{5}$

$\Rightarrow 2 \times \frac{22}{7} r = \frac{11}{5}$

$\Rightarrow r = \frac{7}{20} m$

$= \frac{7 \times 100}{20}$

$= 35\ m$

48. A sector is cut from a circle of diameter 21cm. If the angle of the sector is 150° find its area.

Ans. We have,

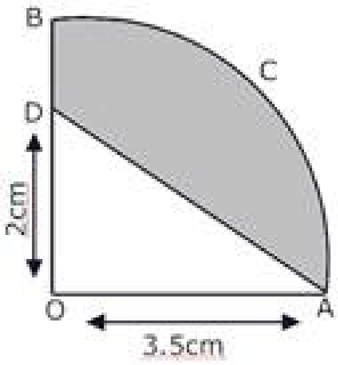
Diameter = 21cm $\Rightarrow$ radius = $\frac{21}{2}$ cm

Angle of sector = $150°$

Area of the sector $= A = \frac{\theta}{360°} \times \pi r^2 = \frac{150°}{360°} \times \frac{22}{7} \times \left(\frac{21}{2}\right)^2$

$= \frac{5}{12} \times \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2} = \frac{5 \times 11 \times 21}{4 \times 2} = 144.38\ cm^2$

49. In the given figure AOBCA represent a quadrant of area 9.625 cm^2. Calculate the area of the shaded portion.



Ans. Required area = Area of quadrant OAB – Areaof ΔOAB

$= \frac{1}{4}\pi(3.5)^2 - \frac{1}{2}(2)(3.5)$

$= \frac{1}{4} . \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} - \frac{7}{2}$

$= \frac{77}{8} - \frac{7}{2} = \frac{77-28}{8}$

$= \frac{49}{8} = 6.125\ cm^2$

CBSE Class 10 Mathematics

Important Questions

Chapter 12

Area Related to Circles

3 Marks Questions

1. The cost of fencing a circular field at the rate of Rs. 24 per metre is Rs. 5280. The field is to be ploughed at the rate of Rs. 0.50 Per m^2. Find the cost of ploughing the field.

$\left[\pi = \frac{22}{7}\right]$

Ans. Since for Rs. 24, the length of fencing = 1metre

$\therefore$ for Rs 5280, the length of fencing $= \frac{1}{24} \times 5280 = 220m$

$\therefore$ Perimeter i.e. circumference of the field = 220m

Let r be radius of the field.

$\therefore 2\pi r = 220$

$\Rightarrow \frac{2 \times 22}{7} r = 220$

$\Rightarrow r = \frac{220 \times 7}{44}$

$= 5 \times 7 = 35m$

$\therefore$ Area of the field = $\pi r^2 = \pi(35)^2 = 1225\pi m^2$

Rate = Rs 0.50 Per m^2

$\therefore$ Total cost of ploughing the field = $Rs\left(1225\pi \times 0.50\right) = Rs\,\frac{1225 \times 22 \times 1}{7 \times 2}$

$= Rs[175 \times 11) = Rs1925$

2. The length of the minute hand of a clock is 14cm. Find the area swept by the minute hand in 5 minutes.

Ans. Angle covered by minute hand in 60 minutes $= 360°$

$\therefore$ Angle covered in 1 minute $= \frac{360}{60} = 6°$

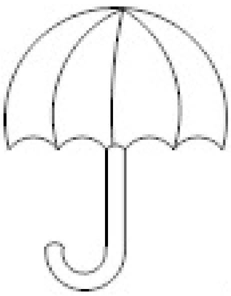
$\therefore$ Angle covered in 5 minutes = $6° \times 5 = 30°$

We know that area swept by the minute hand during this period

=Area of sector with sector angle $30°$

$$= \frac{\theta}{360}(\pi r^2) = \frac{30}{360} \times \frac{22}{7} \times 14 \times 14$$
$$= \frac{1}{12} \times \frac{22}{7} \times 14 \times 14 = \frac{22 \times 7}{3}$$
$$= 51\frac{1}{3} cm^2$$

3. An umbrella has 8 ribs which are equally spaced as given in the figure. Assuming umbrella to be a flat circle of radius 45cm. Find the area between two consecutive ribs of the umbrella.



Ans. Area between two consecutive ribs of the umbrella

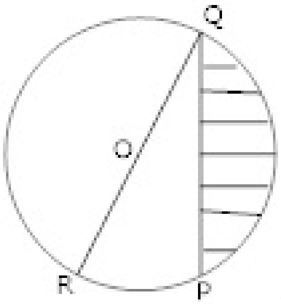
= Area of the sector with angle $\frac{360}{8}$ made at centre

$= \frac{360/8}{360} \pi (45)^2$

$= \frac{1}{8} \times \frac{22}{7} \times (45)^2 = \frac{11}{28} \times 2025 cm^2$

$= 795.58 cm^2 nearly.$

4. Find the area of the shaded region if PQ = 24cm, PR = 7cm and O is the centre of the circle.



Ans. Area of the shaded region

= Area of the semi-circle with O as centre and OQ as radius – area of ΔPQR.......(i)

Since QR is a diameter passing through the centre O of the circle

$\therefore \angle RPQ = 90^\circ$ [Angle of semi-circle]

$\therefore QR^2 = PR^2 + PQ^2 = 7^2 + 24^2 = 49 + 576 = 625 = 25^2$

Þ $QR = 25$

∴ Diameter of the circle = 25cm

And $r = \frac{25}{2} cm$

∴ Area of the semi circle $= \frac{1}{2} \pi r^2$

$$=\frac{1}{2}\times\frac{22}{7}\times\frac{25}{2}\times\frac{25}{2}$$
$$=\frac{11\times625}{28}cm^2$$

Also, area of $\Delta PQR=\frac{1}{2}PR\times PQ$

$$=\frac{1}{2}\times7\times24=7\times12$$
$$=84cm^2$$

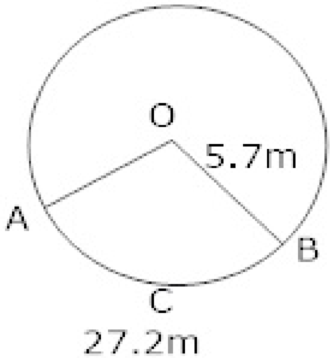
Hence, area of the shaded region

$$=\frac{11\times625}{28}-84=\frac{11\times625-84\times28}{28}$$
$$=\frac{6875-2352}{28}=\frac{4523}{28}$$
$$=126.644\ cm^2$$

5. The perimeter of a sector of a circle of radius 5.7m is 27.2m.Calculate:

(i) The length of arc of the sector in cm.

(ii) The area of the sector in cm^2 correct to the nearest cm^2



Ans. Let O be the centre of a circle of radius 5.7m and OACB be the given sector with perimeter 27.2m.

Then OA=OB=5.7m

Now, $OA + OB + arc\ AB = 27.2m$

$$\Rightarrow 5.7m + 5.7m + arc\ AB = 27.2m$$
$$\Rightarrow arc\ AB = 27.2 - 11.4 = 15.8m$$

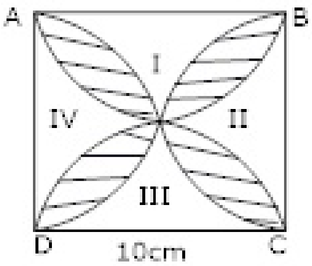
(i) Length of arc AB = 15.8m

(ii) Area of sector $OACB = \left(\frac{1}{2} \times radius \times arc\right)$

$$= \left(\frac{1}{2} \times 5.7 \times 15.8\right) cm^2$$
$$= 45.03 cm^2$$

Area of sector correct to nearest $cm^2 = 45\ cm^2$

6. Find the area of shaded region in the given figure where ABCD is a square of side 10cm and semi-circles are drawn with each side of square as diameter. $[\pi = 3.14]$



Ans. Area of region I + II = area of ABCD – area of 2 semicircles of each radius 5cm

$$= 10 \times 10 - 2 \times \frac{1}{2} \times \pi \times 5^2$$
$$= 100 - 25\pi = 100 - 25 \times 3.14$$
$$= 21.5 cm^2$$

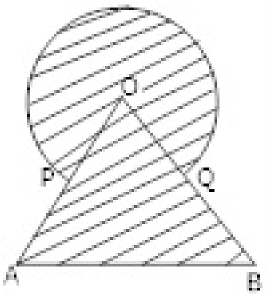
Similarly, area of III + area of IV = $21.5 cm^2$

Area of region I, II, III, and IV $= 2 \times 21.5 = 43 cm^2$

Thus, area of shaded region = Area ABCD – Area of (I, II, III, IV)

$= 100 - 43 = 57 cm^2$

7. Find the area of the shaded region where a circular arc of radius 6cm has been drawn with vertex O of an equilateral triangle OAB of side 12cm as centre.



Ans. Area of shaded region

= Area of major sector OPLQO + Area of equilateral ΔOAB

$$= \frac{300}{360}\pi(6)^2 + \frac{\sqrt{3}}{4}(12)^2$$

$$= \frac{5}{6} \times \frac{22}{7} \times 36 + \frac{\sqrt{3}}{4} \times 144 = \left(\frac{660}{7} + 36\sqrt{3}\right) cm^2$$

8. In the given figure ΔABC is an equilateral triangle inscribed in a circle of radius 4 cm and centre O. Show that the area of the shaded region is $\frac{4}{3}\left(4\pi - 3\sqrt{3}\right) cm^2$.

Ans. In ΔOBD, Let $BD = a$, $OB = 4\,cm$

$$\sin 60° = \frac{BD}{OB}$$

$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{a}{4}$$

$$\Rightarrow a = \frac{4\sqrt{3}}{2} = 2\sqrt{3} cm$$

$$BC = 2a = 2 \times 2\sqrt{3} = 4\sqrt{3} cm$$

$$\frac{OD}{4} = \cos 60^\circ \Rightarrow OD = 4 \times \frac{1}{2} = 2cm$$

$\therefore$ Area of shaded region = Area of sector OBPC – Area of ΔOBC

$$= \frac{120}{360^\circ} \times \pi \times 4^2 - \frac{1}{2} \times 4\sqrt{3} \times 2$$

$$= \frac{4}{3}\left[4\pi - 3\sqrt{3}\right] cm^2$$

9. The radii of two circles are 8cm and 6cm respectively. Find the radius of the circle having its area equal to the sum of the areas of the two circles.

Ans. A_1 = Area of the first circle = $\pi(8)^2 = 64\pi\ cm^2$

A_2 = Area of the second circle = $\pi(6)^2 = 36\pi\ cm^2$

$A_1 + A_2$ = Total area = $64\pi + 36\pi = 100\pi\ cm^2$

Let R be the radius of the circle with area $A_1 + A_2$

$$\therefore \pi R^2 = 100\pi \Rightarrow R^2 = 100 \Rightarrow R = 10\,cm$$

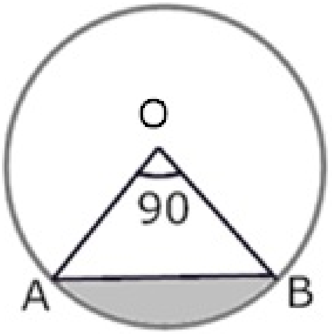
Hence, required radius = 10cm

10. A chord of a circle of radius 10cm subtends a right angle at the centre. Find the area of the corresponding: (Use $\pi = 3.14$)

(i) minor sector

(ii) major sector

(iii) minor segment



(iv) major segment

Ans. (i) Area of minor sector = $\frac{\theta}{360}\pi r^2$

$$= \frac{90}{360} \times 3.14 \times (10)^2$$
$$= \frac{1}{4}(3.14)(100)$$
$$= \frac{314}{4} = 78.50 = 78.5\ cm^2$$

(ii) Area of major sector = Area of circle – Area of minor sector

$$= \pi(10)^2 - \frac{90}{360}\pi(10)^2 = (3.14)(100) - \frac{1}{4}(3.14)(100)$$
$$= 314 - 78.50 = 235.5\ cm^2$$

(iii) We know that area of minor segment

= Area of minor sector OAB – Area of ΔOAB

$$\left[\begin{array}{l} \because \text{ area of } \Delta\text{OAB} = \frac{1}{2}(OA)(OB)\sin\angle AOB \\ = \frac{1}{2}(OA)(OB)(\because \angle AOB = 90^\circ) \\ \text{Area of sector} = \frac{\theta}{360}\pi r^2 \end{array}\right]$$

$$= \frac{1}{4}(3.14)(100) - 50 = 25(3.14) - 50$$

$$= 78.50 - 50 = 28.5 \ cm^2$$

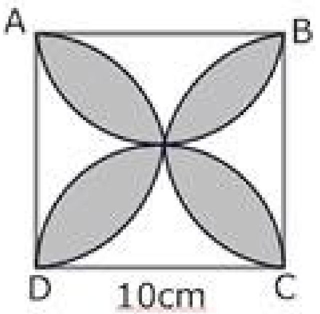
(iv) Area of major segment = Area of the circle – Area of minor segment

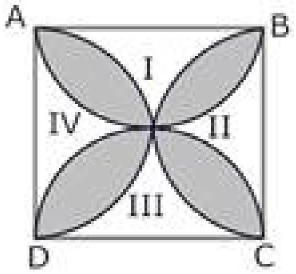
$$= r(10)^2 - 25.5$$

$$= 100(3.14) - 28.5$$

$$= 314 - 28.5 = 285.5 \ cm^2$$

11. Find the area of the shaded region in the given figure where ABCD is a square of side 10cm and semi-circle are drawn with each side of the square as diameter. $(\pi = 3.14)$





Ans. Let us mark the four unshaved regions as I, II, III and IV

Thus, area of I + Area of III

= Area of ABCD – Area of two semi circles of each of radius 5 cm

$$= 10 \times 10 - 2 \times \frac{1}{2} \times \pi \times 5^2$$

$$= 100 - 25\pi = 100 - 25 \times 3.14$$

$$= 100 - 78.50$$

$$= 21.5 \ cm^2$$

Similarly, area of II + area of IV = 21.5cm^2

Hence, area of the unshaded region i.e.,

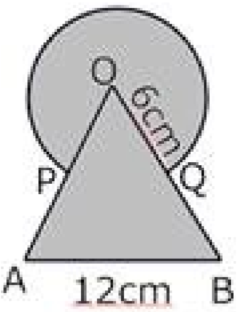
I, II, III, IV

= 2(21.5) cm^2=45cm^2

Thus, area of the shaded region

= Area ABCD – Area of (I,II,III and IV) = 100 – 43= 57cm^2

12. Find the area of the shaded region where a circular arc of radius 6cm has been drawn with vertex O of an equilateral triangle OAB of side 12cm as centre.



Ans. Area of the shaded region

= Area of major sector OPLQO + Area of equilateral triangle OAB

$$=\frac{300}{360}\pi(6)^2+\frac{\sqrt{3}}{4}(12)^2\left[\begin{array}{l}\because \text{ area of equilateral triangle with side} \\ a=\frac{\sqrt{3}}{4}a^2 \text{ and } \angle POQ-300^\circ\end{array}\right]$$

$$=\frac{5}{6}\times\frac{22}{7}\times 36\times\frac{\sqrt{3}}{4}\times 144$$

$$=\left(\frac{660}{7}+36\sqrt{3}\right)cm^2$$

13. In Akshita’s house, there is a flower pot. The sum of radii of circular top and bottom of a flowerpot is 140 cm and the difference of their circumference is 88cm, find the

diameter of the circular top and bottom.

Ans. Sum of radii of circular top and bottom = 140cm

Let radius of top = r cm

$\therefore$ rRadius of bottom = $(140-r)cm$

Circumference of top = $2\pi r\ cm$

Circumference of bottom = $2\pi(140-r)cm$

Difference of circumference = $[2\pi r-2\pi(140-r)]cm$

By the given condition,

$$2\pi r-2\pi(140-r)=88$$

$$\Rightarrow 2\pi[r-140+r]=88$$

$$\Rightarrow 2r-140=\frac{88}{2\pi}=\frac{44}{\frac{22}{7}}=14$$

$$\Rightarrow 2r=140+14-154$$

$$\Rightarrow r=\frac{154}{2}=77$$

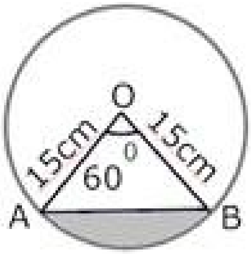
$\therefore$ Radius of top = 77cm

$\Rightarrow$ Diameter of top = $2\times 77=154\ cm$

Radius of bottom = 140 – r=140 – 77=63cm

$\Rightarrow$ Diameter of bottom = 2×63 = 126cm

14. A chord of a circle of radius 15cm subtends an angle of 60° at the centre. Find the area of the corresponding minor and major segments of the circle. (use $\pi=3.14$ and $\sqrt{3}=1.73$)



Ans. We know that area of minor segment

= Area of minor sector OAB – area of ΔOAB

$$= \frac{60}{360}\pi(15)^2 - \frac{\sqrt{3}}{4}(15)^2 = \frac{1}{6}(3.14)(225) - \frac{(1.73)}{4}(225)$$

$[\because \Delta OAB \text{ is an equilaterl triangle}]$

$$= (1.57)(75) - \frac{(1.73)(225)}{4}$$

$$= 117.75 - 97.3125 = 20.4375\ cm^2$$

Again, area of major segment = Area of the circle – area of minor segment

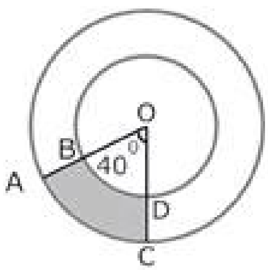
$$= r(15)^2 - 20.4375$$

$$= (3.14)(225) - 20.4375$$

$$= 706.5 - 20.4375$$

$$= 686.0625\ cm^2$$

15. Find the area of the shaded region of the two concentric circles with centre O and radii 7cm and 14 cm respectively and $\angle AOC = 40°$.



Ans. Area of the shaded region

= area of sector OAC – area of sector OBD

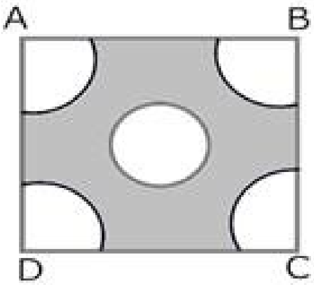
$$= \frac{4}{360}\pi(14)^2 - \frac{40}{360}\pi(7)^2 = \frac{\pi}{9}(196-49)$$

$$= \frac{22}{7\times 9}147 = \frac{22\times 21}{9} = \frac{22\times 7}{3}$$

$$= \frac{154}{3}cm^2$$

$$= 51\frac{1}{3}cm^2$$

16. From each corner of a square of side 4cm, a quadrant of a circle of radius 1 cm is cut and also a circle of diameter 2cm is cut as shown in the figure. Find the area of the remaining portion of the square.



Ans. Area of the shaded region = Area of square ABCD -4 (area of quadrant of acircle with radius 1) - $\pi\ (1)^2$

$$= 4\times 4 - 4.\frac{1}{4}\pi(1)^2 - \pi(1)^2 = 16 - \pi - \pi = 16 - 2\pi$$

($\because$ Radius of the circle = 1cm)

$$= 16 - \frac{2\times 22}{7} = \frac{112-44}{7} = \frac{68}{7}cm^2$$

17. The length of the minute hand of clock is 14cm. Find the area swept by the minute hand is 5 minutes.

Ans. Angle covered by minute hand in 60 minutes = $360°$

$\therefore$ Angle covered in 1 minute = $\frac{360}{60} = 6°$

$\therefore$ Angle covered in 5 minutes = $6° \times 5 = 30°$

We know that area swept by the minute hand during this period

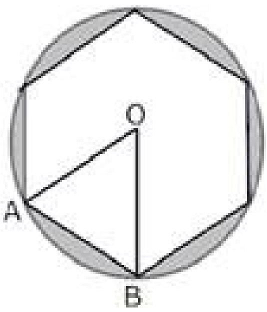
= Area of the sector with sector angle $30°$

$= \frac{\theta}{360}(\pi r^2) = \frac{30}{360} \times \frac{22}{7} \times 14 \times 14 \quad [\therefore r = 14\ cm]$

$= \frac{22}{12} \times 28 = \frac{22 \times 7}{3}$

$= \frac{154}{3} = 51\frac{1}{3}\ cm^2$

18. A round table cover has six equal designs as shown in the figure. If the radius of the cover is 28cm, find the cost of making the design at the rate of Rs.0.35 per cm^2. (Use $\sqrt{3} = 1.7$)



Ans. Area of one design = $\frac{60}{360} \times \pi(28)^2$ – Area of ΔOAB

$= \frac{\pi}{6} \times (28)^2$ – Area of equilateral ΔOAB

$= \frac{\pi}{6} \times (28)^2 - \frac{\sqrt{3}}{4}(28)^2$

$= (28)^2\left(\frac{\pi}{6} - \frac{\sqrt{3}}{4}\right)$

[Area of equilateral triangle with side a = $\frac{\sqrt{3}}{4}a^2$]

$$=(28)^2\left(\frac{22}{7\times}-\frac{1.7}{4}\right)$$

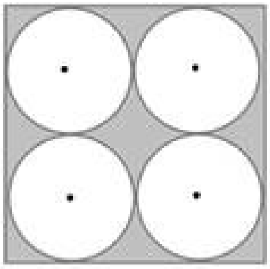
Total cost = $6\times(28)^2\left(\frac{22}{42}-\frac{17}{40}\right)\times 0.35$

$$=21\times(28)^2\left(\frac{11}{21}-\frac{17}{40}\right)=\frac{21\times(28)^2}{100}\times\frac{440-357}{21\times 40}$$

$$=\frac{28\times 28\times 83}{40}=\frac{7\times 28\times 83}{1000}$$

= $Rs.16.268$

19. Find the area of the shaded region where ABCD is a square of side 14cm.



Ans. Area of square ABCD = $14\times 14cm^2=196cm^2$

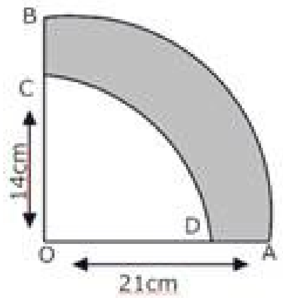
Diameter of each circle = $\frac{14}{2}=7cm$

∴ Radius of each circle = $\frac{7}{2}cm$

∴ Area of one circle = $\pi r^2=\frac{22}{7}\times\frac{7}{2}\times\frac{7}{2}cm^2=\frac{154}{4}=\frac{77}{2}cm^2$

Area of the four circles = $4\times\frac{77}{2}cm^2=154\ cm^2$

20. ABCD is a flower bed. If OA = 21m and DC = 14m. Find the area of the bed.



Ans. Here, OA = R = 21m and OC = r = 14m

∴ Area of the flower bed (i.e., shaded portion)

= area of quadrant of a circle of radius R of the quadrant of a circle of radius r

$$= \frac{1}{4}\pi R^2 - \frac{1}{4}\pi r^2 = \frac{\pi}{4}\left(R^2 - r^2\right)$$

$$= \frac{1}{4} \times \frac{22}{7}\left[(21)^2 - (14)^2\right] m^2 \quad [\because R = 21\ m \text{ and } r = 14\ m]$$

$$= \frac{1}{4} \times \frac{22}{7} \times (21 \times 14)(21 - 14)\, m^2$$

$$= \frac{1}{4} \times \frac{22}{7} \times 35 \times 7 m^2$$

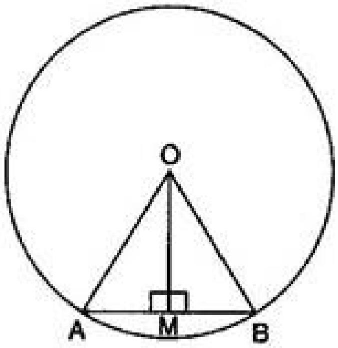
$$= 192.5\ m^2$$

21. In a circle of radius 21 cm, an arc subtends an angle of 60° at the centre. Find:

(i) the length of the arc.

(ii) area of the sector formed by the arc.

(iii) area of the segment formed by the corresponding chord



Ans. Given, r = 21 cm and $\theta = 60^\circ$

(i) Length of arc = $\frac{\theta}{360^\circ} \times 2\pi r$ = $\frac{60^\circ}{360^\circ} \times 2 \times \frac{22}{7} \times 21$ = 22 cm

(ii) Area of the sector = $\frac{\theta}{360^\circ} \times \pi r^2$ = $\frac{60^\circ}{360^\circ} \times \frac{22}{7} \times 21 \times 21$ = 231 cm^2

(iii) Area of segment formed by corresponding chord

= $\frac{\theta}{360^\circ} \times \pi r^2$ – Area of ΔOAB

$\Rightarrow$ Area of segment = 231 – Area of ΔOAB ....(i)

In right angled triangle OMA and OMB,

OM = OB [Radii of the same circle]

OM = OM [Common]

$\therefore$ ΔOMA $\cong$ ΔOMB [RHS congruency]

$\therefore$ AM = BM [By CPCT]

$\therefore$ M is the mid-point of AB and $\angle$ AOM = $\angle$ BOM

$\Rightarrow$ $\angle$ AOM = $\angle$ BOM = $\frac{1}{2}\angle$AOB = $\frac{1}{2} \times 60^\circ = 30^\circ$

Therefore, in right angled triangle OMA,

$$\cos 30^\circ = \frac{OM}{OA}$$

$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{OM}{21}$$

$$\Rightarrow OM = \frac{21\sqrt{3}}{2} \text{ cm}$$

Also, $\sin 30^\circ = \frac{AM}{OA}$

$$\Rightarrow \frac{1}{2} = \frac{AM}{21}$$

$$\Rightarrow AM = \frac{21}{2} \text{ cm}$$

$\therefore$ AB = 2 AM = $2 \times \frac{21}{2}$ = 21 cm

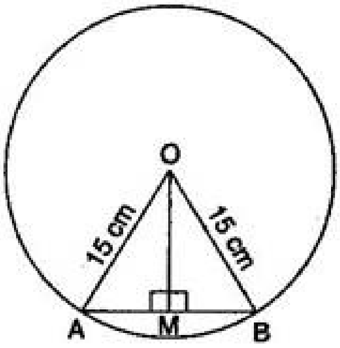
$\therefore$ Area of ΔOAB = $\frac{1}{2}$ x AB x OM

$= \frac{1}{2} \times 21 \times \frac{21\sqrt{3}}{2} = \frac{441\sqrt{3}}{4} \text{ cm}^2$

Using eq. (i),

Area of segment formed by corresponding chord = $\left(231 - \frac{441\sqrt{3}}{4}\right) \text{cm}^2$

22. A chord of a circle of radius 15 cm subtends an angle of 60° at the centre. Find the area of the corresponding segment of the circle. $\left(\text{Use } \pi = 3.14 \text{ and } \sqrt{3} = 1.73\right)$



Ans. Here, r = 15 cm and $\theta = 60^\circ$

Area of minor sector = $\frac{\theta}{360^\circ} \times \pi r^2$

= $\frac{60^\circ}{360^\circ} \times 3.14 \times 15 \times 15$

= 117.75 cm^2

For, Area of $\triangle$AOB,

Draw OM $\perp$ AB.

In right triangles OMA and OMB,

OA = OB [Radii of same circle]

OM = OM [Common]

$\therefore$ $\triangle$OMA $\cong$ $\triangle$OMB [RHS congruency]

$\therefore$ AM = BM [By CPCT]

$\Rightarrow$ AM = BM = $\frac{1}{2}$ AB and $\angle$AOM

= $\angle$BOM = $\frac{1}{2}\angle$AOB = $\frac{1}{2} \times 60^\circ = 30^\circ$

In right angled triangle OMA,

$\cos 30^\circ = \frac{OM}{OA}$

$\Rightarrow \frac{\sqrt{3}}{2} = \frac{OM}{15}$

$\Rightarrow$ OM = $\frac{15\sqrt{3}}{2}$ cm

Also, $\sin 30^\circ = \frac{AM}{OA}$

$\Rightarrow \frac{1}{2} = \frac{AM}{15}$

$\Rightarrow$ AM = $\frac{15}{2}$ cm

$\Rightarrow$ 2 AM = $2 \times \frac{15}{2}$ = 15 cm

$\Rightarrow$ AB = 15 cm

$\therefore$ Area of ΔAOB = $\frac{1}{2}$ x AB x OM

= $\frac{1}{2} \times 15 \times \frac{15\sqrt{3}}{2}$ = $\frac{225\sqrt{3}}{3}$

= $\frac{225 \times 1.73}{4}$ = 97.3125 cm^2

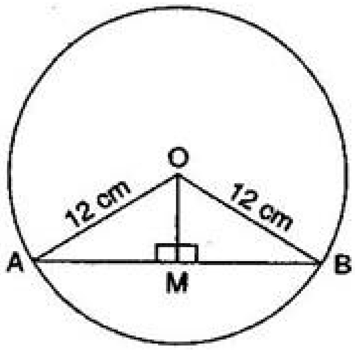
$\therefore$ Area of minor segment = Area of minor sector – Area of ΔAOB

= 117.75 – 97.3125 = 20.4375 cm^2

And, Area of major segment = πr^2 – Area of minor segment

= 706.5 – 20.4375 = 686.0625 cm^2

23. A chord of a circle of radius 12 cm subtends an angle of 120° at the centre. Find the area of the corresponding segment of the circle. $\left(\text{Use } \pi = 3.14 \text{ and } \sqrt{3} = 1.73\right)$



Ans. Here, r = 15 cm and $\theta = 120^\circ$

Area of corresponding sector = $\frac{\theta}{360^\circ} \times \pi r^2$

= $\frac{120^\circ}{360^\circ} \times 3.14 \times 12 \times 12$

= 150.72 cm^2

For, Area of ΔAOB,

Draw OM $\perp$ AB.

In right triangles OMA and OMB,

OA = OB [Radii of same circle]

OM = OM [Common]

$\therefore$ ΔOMA $\cong$ ΔOMB [RHS congruency]

$\therefore$ AM = BM [By CPCT]

$\Rightarrow$ AM = BM = $\frac{1}{2}$ AB and $\angle$AOM = $\angle$BOM

= $\frac{1}{2}\angle$AOB = $\frac{1}{2} \times 120^\circ = 60^\circ$

In right angled triangle OMA,

$$\cos 60^\circ = \frac{OM}{OA}$$

$$\Rightarrow \frac{1}{2} = \frac{OM}{12}$$

$\Rightarrow$ OM = 6 cm

Also, $\sin 60^\circ = \frac{AM}{OA}$

$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{AM}{12}$$

$\Rightarrow$ AM = $6\sqrt{3}$ cm

$\Rightarrow$ 2 AM = $2 \times 6\sqrt{3}$ = $12\sqrt{3}$ cm

$\Rightarrow$ AB = $12\sqrt{3}$ cm

$\therefore$ Area of ΔAOB = $\frac{1}{2}$ x AB x OM

= $\frac{1}{2} \times 12\sqrt{3} \times 6$ = $36\sqrt{3}$

= 36 x 1.73 = 62.28 cm^2

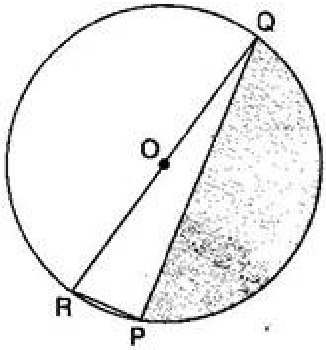
$\therefore$ Area of corresponding segment = Area of corresponding sector – Area of ΔAOB

= 150.72 – 62.28 = 88.44 cm^2

Unless stated otherwise, take $\pi = \frac{22}{7}$.

24. Find the area of the shaded region in figure, if PQ = 24 cm, PR = 7 cm and O is the

centre of the circle.



Ans. $\angle$RPQ = 90° [Angle in semi circle is 90°]

$\therefore$ $RQ^2 = PR^2 + PQ^2 = (7)^2 + (24)^2$ = 49 + 576 = 625

$\Rightarrow$ RQ = 25 cm

$\Rightarrow$ Diameter of the circle = 25 cm

$\therefore$ Radius of the circle = $\frac{25}{2}$ cm

Area of the semicircle = $\frac{1}{2}\pi r^2 = \frac{1}{2} \times \frac{22}{7} \times \frac{25}{2} \times \frac{25}{2}$

= $\frac{6875}{28}$ cm^2

Area of right triangle RPQ = $\frac{1}{2}$ x PQ x PR

= $\frac{1}{2} \times 24 \times 7$ = 84 cm^2

Area of shaded region = Area of semicircle – Area of right triangle RPQ

= $\frac{6875}{28} - 84 = \frac{6875 - 2352}{28}$

= $\frac{4523}{28}$ cm^2

CBSE Class 10 Mathematics

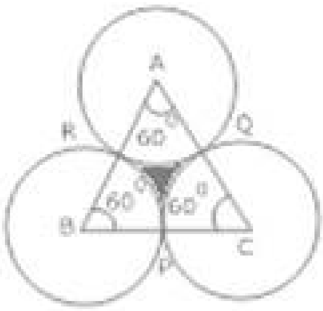
Important Questions

Chapter 12

Area Related to Circles

4 Marks Questions

1. The area of an equilateral ΔABC is 1732.5 cm^2 with each centre of the triangle as vertex of circle is drawn with radius equal to half the length of the side of triangle, find the area of the shaded region. $\left[\pi = 3.14 \ \sqrt{3} = 1.73205\right]$



Ans. Area of shaded region = area of $\Delta ABC - 3$ (Area of sector BPR)

Let 'a' be the side of the equilateral Δ

$\therefore \frac{\sqrt{3}}{4}a^2 = 17320.5 = \frac{1.73205}{4}a^2 = 17320.5$

$\Rightarrow a^2 = \frac{4 \times 17320.5}{1.73205} = 4 \times 10000$

$\Rightarrow a = 2 \times 100 = 200$

$\therefore$ Required area = $17320.5 - 3\left[\frac{60}{360} \times \pi (100)^2\right]$

$= 17320.5 - \frac{1}{2} \times 3.14 \times 10000$

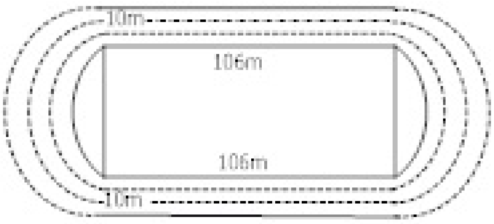
$= 17320.5 - 1.57 \times 10000$

$= 17320.5 - 15700 = 1620.5 cm^2$

2. The given figure depicts a racing track whose left and right ends are semi-circular. The difference between the two inner parallel line segments is 60m and they are each 106m long. If the track is 10m wide, find:

(i) The distance around the track along its inner edge,

(ii) The area of the track.



Ans. (i) The distance around the track along the inner edge

$$=106+106+(\pi\times 30+\pi\times 30)$$

$$=212+\frac{22}{7}\times 60=212+\frac{1320}{7}$$

$$=\frac{2804}{7}m$$

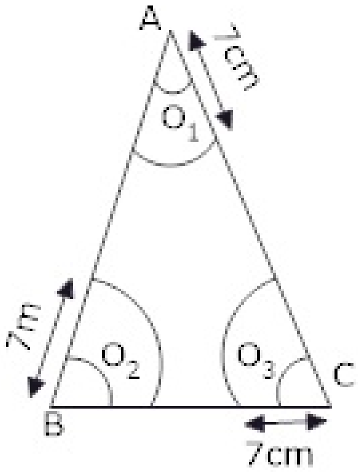
(ii) The area of the track $=106\times 80-106\times 60+2.\frac{1}{2}\pi\left[40^2-30^2\right]$

$$=106\times 20+\pi(70)(10)$$

$$=2120+700\times\frac{22}{7}=2120+2200$$

$$=4320m^2$$

3. Three horses are tethered with 7m long ropes at the three corners of a triangular field having sides 20m, 34m and 42 m. Find the area of the plot which can be grazed by the horses. Also, find the area of the plot which remains ungrazed.



Ans. Let $\angle A = \theta_1°, \angle B = \theta_2° and \angle C = \theta_3°$

Area which can be grazed by three horses = Area of sector with central angle $\theta_1°$ and radius 7 cm + Area of sector with central angle $\theta_2°$ and radius 7 cm + Area of sector with central angle $\theta_3°$ and radius 7 cm

$$= \frac{\pi r^2 \theta_1°}{360} + \frac{\pi r^2 \theta_2°}{360°} + \frac{\pi r_2 \theta_3°}{360°}$$

$$= \frac{\pi r^2}{360°}(\theta_1° + \theta_2° + \theta_3°)$$

$$= \frac{\pi r^2}{360°} \times 180$$

($\because$ Sum of three angles of a $\Delta = 180°$)

$$= \frac{22}{7} \times \frac{7 \times 7 \times 180°}{360°} = 77 m^2$$

Sides of plot ABC are = 20m, b = 34m and c = 42m

$\because$ Semi perimeter, $s = \dfrac{20+34+42}{2} = 48\ m$

$\therefore$ Area of triangular plot

= Area of $\Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{48 \times 28 \times 14 \times 6}$

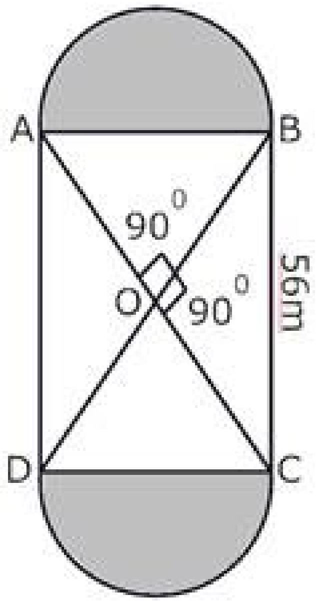
$= 336 m^2$

Area grazed by the horses = $77 m^2$

∴ Ungrazed area = (336 – 77)

= $259\ m^2$

4. In given figure, two circular flower beds have been shown on two sides of a square lawn ABCD of side 56m. If the corner of each circular flower bed is the point of enter section O of the diagonals of the square lawn. Find the sum of the areas of the lawn and the flower beds.



Ans. Area of the square lawn ABCD = $56 \times 56\, m^2$......(i)

Let OA = OB = x meter

∴ By Pythagoras theorem,

$x^2 + x^2 = 56^2 \Rightarrow 2x^2 = 56 \times 56 \Rightarrow x^2 = 28 \times 56$........(ii)

Again, area of sector OAB = $\frac{90}{360}\times\pi x^2 = \frac{1}{4}\pi r^2$

$$=\frac{1}{4}\times\frac{22}{7}\times 28\times 56\ m^2 ....(iii)[From(ii)]$$

Also, area of $\Delta OAB - \frac{1}{4}56\times 56 m^2$

[Here, $\angle AOB = 90°$. Since square ABCD is divided into 4 right triangles]

$\therefore$ Area of flowerbed $AB - \left(\frac{1}{4}\times\frac{22}{7}\times 28\times 56 - \frac{1}{4}56\times 56\right)m^2\ (from 3 \& 4) ....(iv)$

$$=\frac{1}{4}\times 28\times 56\left(\frac{22}{7}-2\right)$$

$$=\frac{7\times 56\times 8}{7}$$

$$=448m^2 ....(v)$$

Similarly, area of the other flowerbed = 448m^2

$\therefore$ Total area of the lawn and the flowerbeds

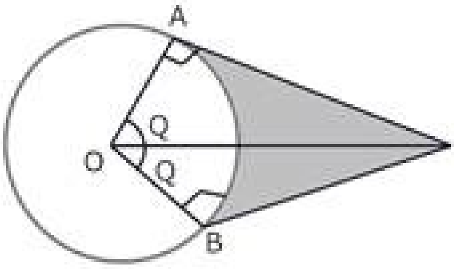
= $56\times 56 + 448 + 448$

$$= 3136 + 896$$

$$= 4032\ m^2$$

5. An elastic belt is placed round the rein of a pulley of radius 5cm. One point on the belt is pulled directly away from the centre O of the pulley until it is at P, 10cm from O. Find the length of the best that is in contact

with the rim of the pulley. Also find the shaded area.



Ans. $\cos\theta = \frac{OA}{OP} = \frac{5}{10} = \frac{1}{2}$

$\Rightarrow \theta = 60^\circ$

$\Rightarrow \angle AOB = 20 = 120^\circ$

$\therefore$ Arc $AB = \frac{120 \times 2 \times \pi \times 5}{360} cm$

$= \frac{10\pi}{3} cm \left[\because l = \frac{\theta}{360} \times 2\pi r\right]$

Length of the belt that is in contact with the rim of the pulley

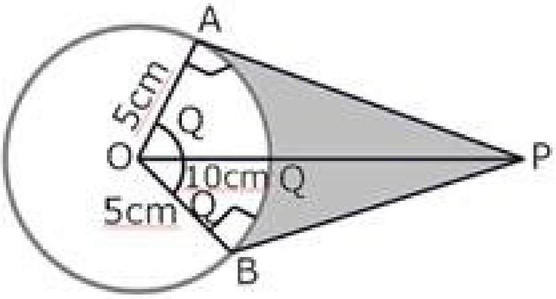
= Circumference of the rim – length of arc AB

$= 2\pi \times 5cm - \frac{10}{3}\pi cm$

$= \frac{20}{3}\pi cm$

Now, area of sector OAQB = $\frac{120}{360} \times \pi \times 5^2\, cm = \frac{25}{3}\pi cm^2$

$\left[\text{since Area } = \frac{\theta}{360} \times \pi r^2\right]$



Area of quadrilateral OAPB

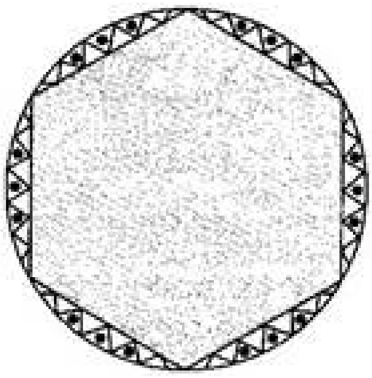
$= 2(\text{Area of } \Delta\text{OAP}) = 25\sqrt{3}cm^2$

$\left[\because AP = \sqrt{100-25} = \sqrt{75} = 5\sqrt{3}cm\right]$

Hence, shaded area $= 25\sqrt{3} - \frac{25}{3}\pi$

$= \frac{25}{3}\left(3\sqrt{3} - \pi\right)cm^2$

6. A round table cover has six equal designs as shown in figure. If the radius of the cover is 28 cm, find the cost of making the designs at the rate of Ts. 0.35 per cm^2. $\left(\text{Use } \sqrt{3} = 1.7\right)$



Ans. r = 28 cm and $\theta = \frac{360^\circ}{6} = 60^\circ$

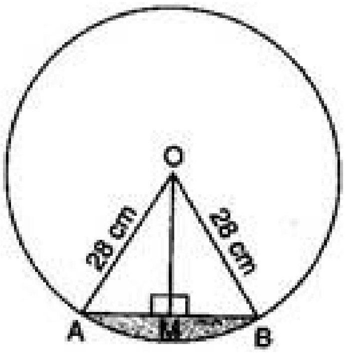
Area of minor sector = $\frac{\theta}{360^\circ} \times \pi r^2$

$= \frac{60^\circ}{360^\circ} \times \frac{22}{7} \times 28 \times 28$

$= \frac{1232}{3}$ = 410.67 cm^2

For, Area of $\triangle$AOB,

Draw OM $\perp$ AB.



In right triangles OMA and OMB,

OA = OB [Radii of same circle]

OM = OM [Common]

$\therefore$ $\triangle$OMA $\cong$ $\triangle$OMB [RHS congruency]

$\therefore$ AM = BM [By CPCT]

$\Rightarrow$ AM = BM = $\frac{1}{2}$ AB and $\angle$AOM

= $\angle$BOM = $\frac{1}{2}\angle$AOB = $\frac{1}{2} \times 60^\circ = 30^\circ$

In right angled triangle OMA, $\cos 30^\circ = \frac{OM}{OA}$

$\Rightarrow \frac{\sqrt{3}}{2} = \frac{OM}{28}$

$\Rightarrow$ OM = $14\sqrt{3}$ cm

Also, $\sin 30^\circ = \frac{AM}{OA}$

$\Rightarrow \frac{1}{2} = \frac{AM}{28}$

$\Rightarrow$ AM = 14 cm

$\Rightarrow$ 2 AM = 2 x 14 = 28 cm

$\Rightarrow$ AB = 28 cm

$\therefore$ Area of ΔAOB = $\frac{1}{2}$ x AB x OM

= $\frac{1}{2} \times 28 \times 14\sqrt{3}$ = $196\sqrt{3}$

= 196×1.7 = 333.2 cm^2

$\therefore$ Area of minor segment = Area of minor sector – Area of ΔAOB

= 410.67 – 333.2 = 77.47 cm^2

$\therefore$ Area of one design = 77.47 cm^2

$\therefore$ Area of six designs = 77.47 x 6 = 464.82 cm^2

Cost of making designs = 464.82 x 0.35 = Rs. 162.68